Tracing the Skyline

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Task

Using a compass (used for measuring horizontal angles) and Abney level (used for measuring vertical angles), estimate the elevations of the mountains surrounding Keswick.

Context

Mountains hold special scientific, cultural, and economic significance. Throughout history, they have been used to demarcate the boundaries between regions. More recently, **triangulation** from mountain summits has allowed us to estimate the size and dimensions of countries with much greater accuracy and precision e.g. the Principal Triangulation of Great Britain (1791 – 1853), as shown below:

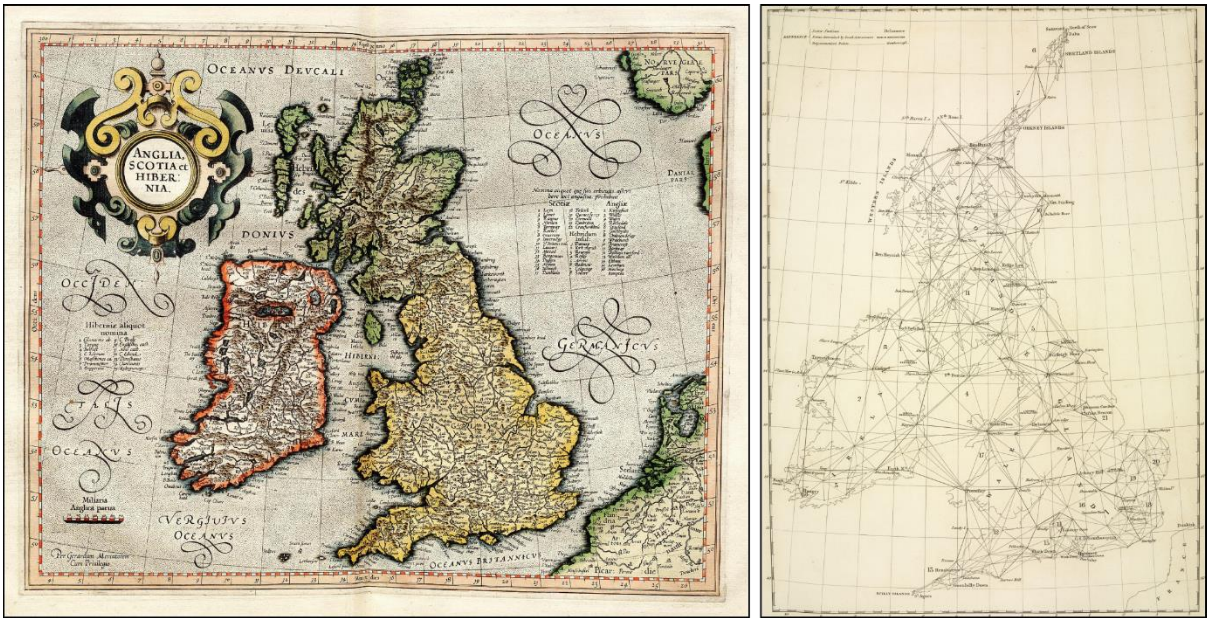


Figure 1: Maps of Great Britain and Ireland before triangulation (**left**: Atlas Cosmographicae by Gerardus Mercator, 1512 – 1594) and after triangulation (**right**: Principal Triangulation of Britain, 1791 – 1853). While triangulation was used for local mapmaking prior to this date, with the earliest evidence for the technique dating to ~1000 AD, this was the first attempt to apply the technique to an entire country.

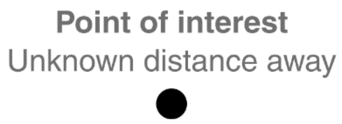
While mountains, coastlines and land areas are measured today using GPS and satellites, triangulation is still used for many purposes, including surveying, navigation, metrology etc. It is also an important component of **Geographic Information Science** (GIS).

Finding the distance to objects

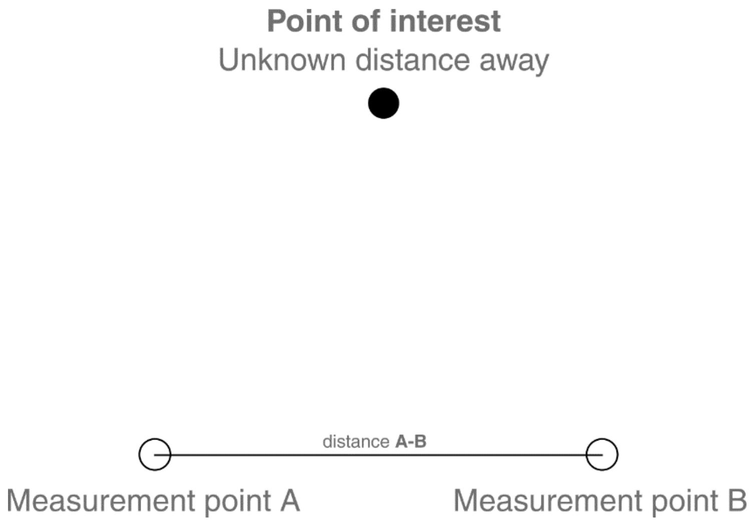
We can calculate the height of distant objects using **trigonometry**.

However, we first need to know the distance to them. This can be achieved by measuring the horizontal angles to that object from two (or more) other measurement points.

For example, let's say we are interested in a particular feature of the landscape (e.g. a mountain), but do not know how far away it is or its elevation. Here, it is represented as a single point:

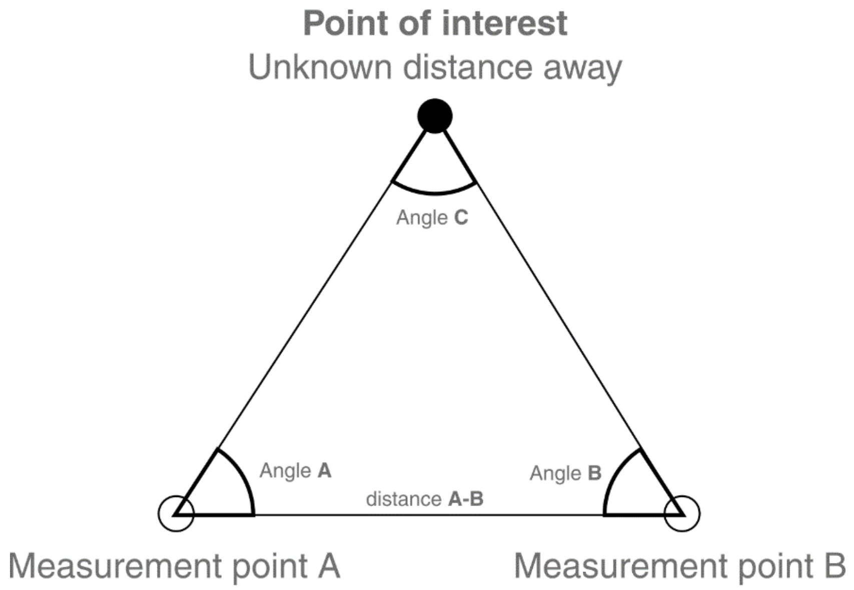


Next, we can select two measurement points (A and B) where we know the distance between them, as shown here:

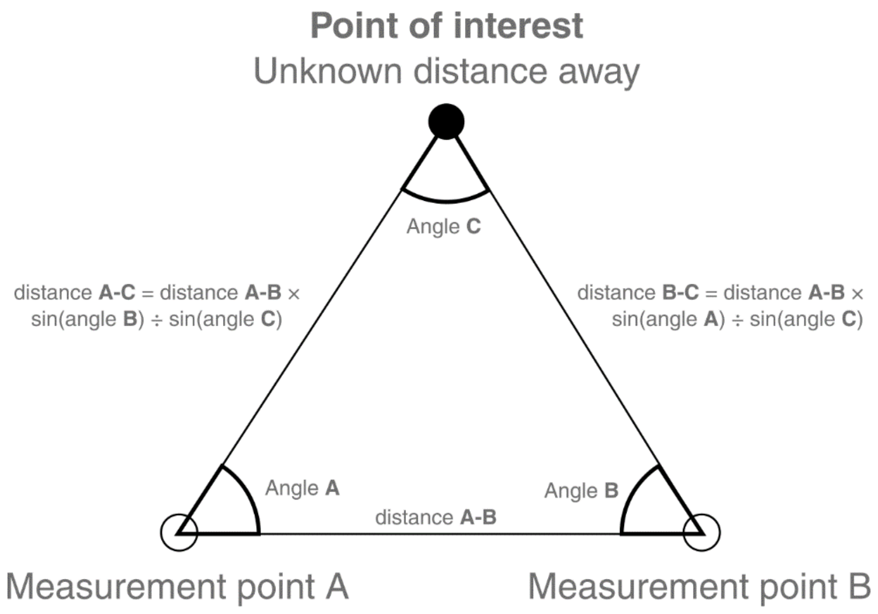


With this information, we now have everything we need to calculate the distance to the unknown point (and in future, it's height).

Our first step is to measure the horizontal angles from A and B to the measurement point using a compass. The final angle of the triangle (Angle **C**) equals 180 – Angle **A** – Angle **B**, because the interior angles of a triangle = 180°.



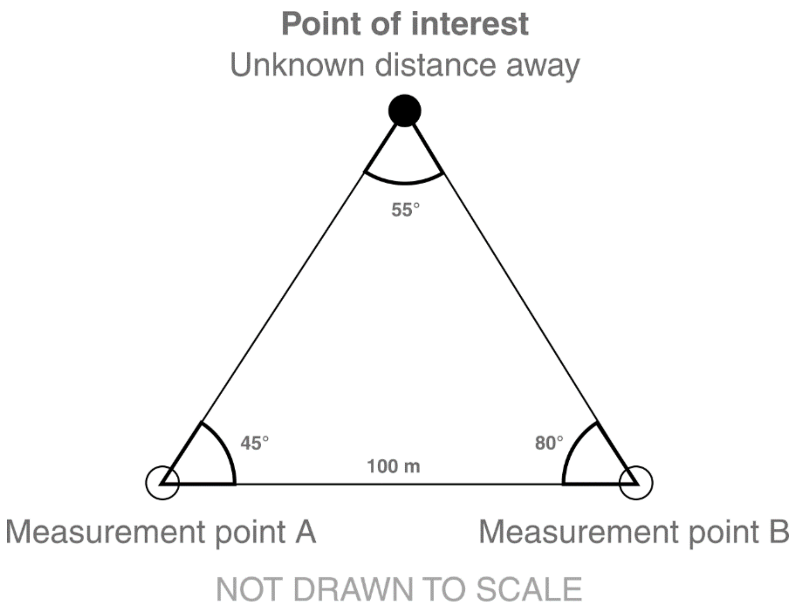
Using this information, the distances between A-C and B-C can be calculated using the trigonometric formulas shown below:



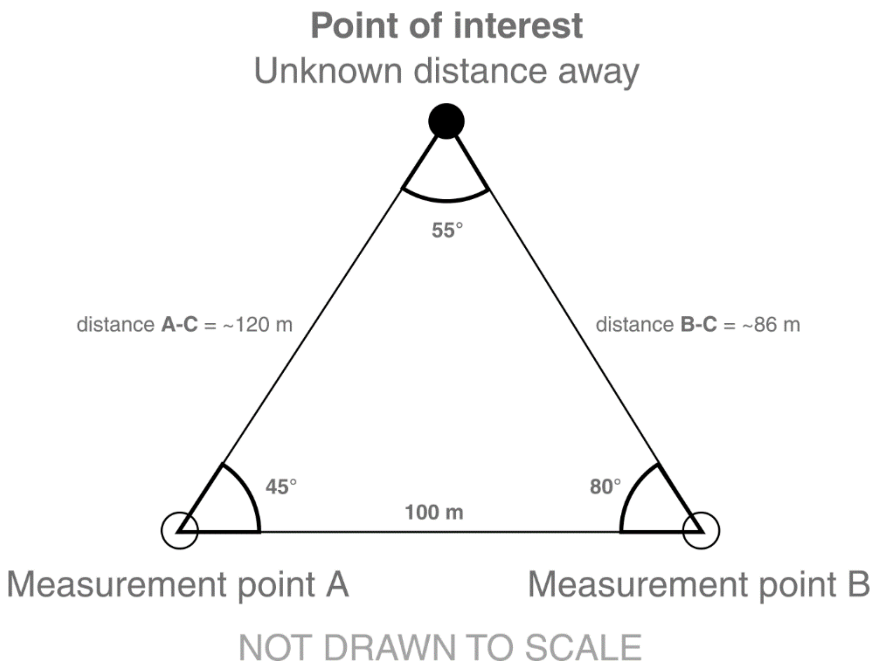
Lots of maths here, but **don't be alarmed**. We'll walk through a simple example on the next page.

Distance calculation: an example

As before, we are interested in a point of interest an unknown distance away and have selected two measurement points (A, B), which are 100 m apart. We have measured the angles from those points to the point of interest (45°, 80°), which leaves a final angle of 55°:



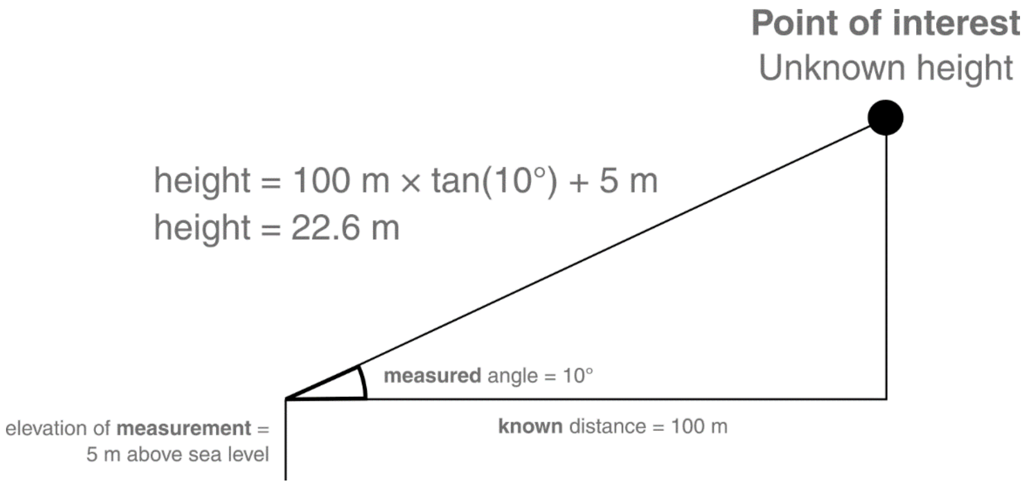
Using the formulas shown on the previous page (e.g. distance **B-C** = distance **A-B** × sin(angle **A**) ÷ sin(angle **C**)), the side lengths are returned as approximately 120 m and 86 m.



Finding the height of objects

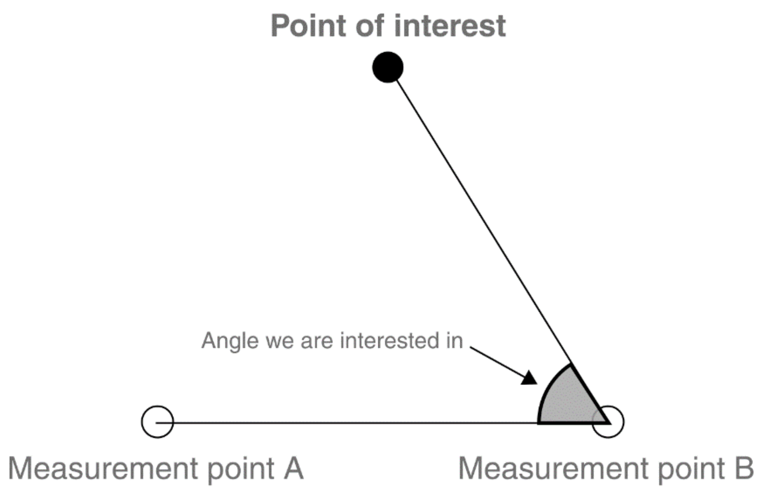
Once we know the distance to an object, finding its height is comparatively easy. This involves measuring the vertical angle to it (using the Abney level) and using the following formula:

Height = distance × tan(angle) + elevation of the observer

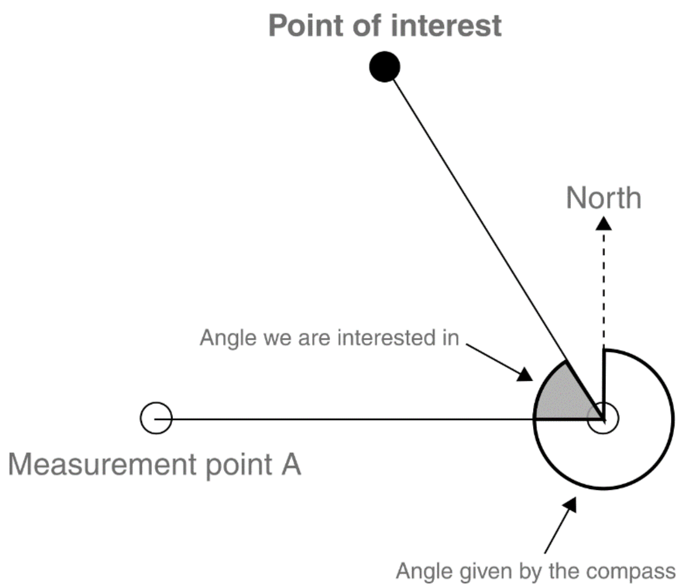


Compass directions vs. angles

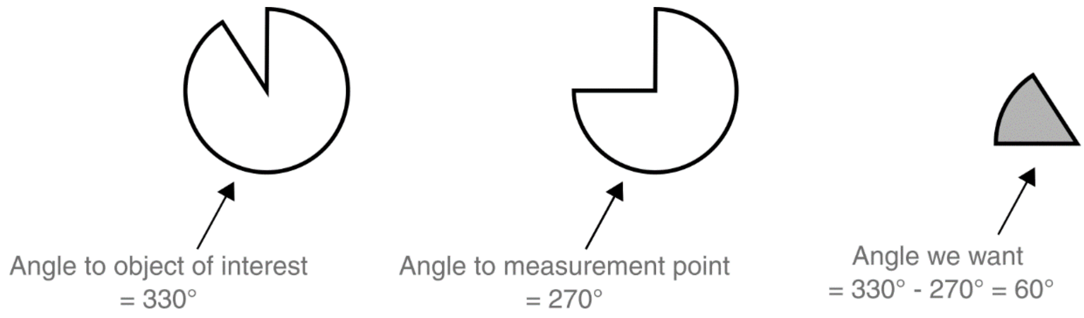
One difficult aspect of this technique is that the directions generated by a compass are not the same as the interior angles of a triangle. When we are measuring the directions, we are actually interested in the angle between our point of interest and the other measurement point, as shown below:



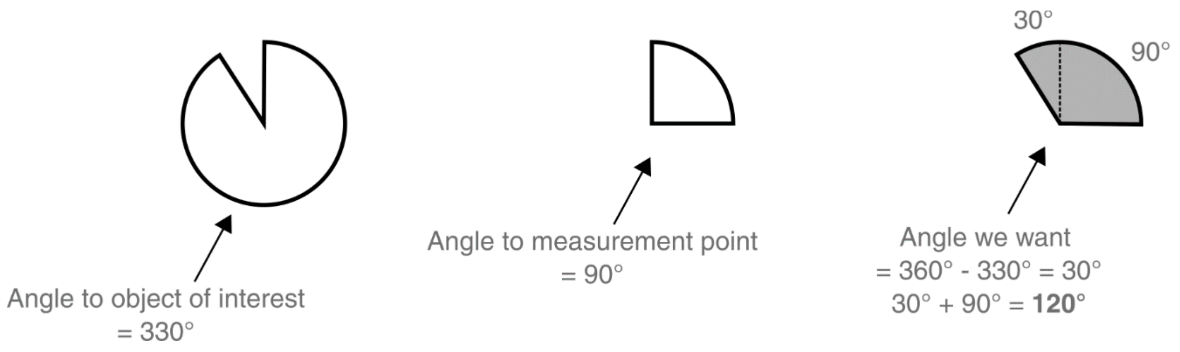
However, angles given by the compass are relative to magnetic north, so what we are actually measuring is the following:



We can retrieve the angle we are interested in if we know the directions between measurement points A and B (for this activity, these are provided for you). Based on the example above, the angle we need is simply the difference between the other two angles:



For other scenarios, including the reverse of the above, you will have to replicate the following:



Visualising the problem (as above) makes it much simpler to understand.

Putting theory into practice

With all this information at your disposal, your **task for today** is to visit five measurement points distributed across Keswick, and use a mirror compass and Abney level to estimate the elevations of the surrounding mountains.

Measurement points are denoted as **A, B, C, D** and **E** and are shown on the map on the following page. You will be given a GPS to locate the exact locations.

At each point, you'll need to measure the directions (compass) and vertical angles (Abney level) to nearby mountain summits. These are numbered **I, II, III, IV** and **V**.

Note, you don't need to take measurements for each summit from every measurement point, only those which are visible and which are listed on the following pages.

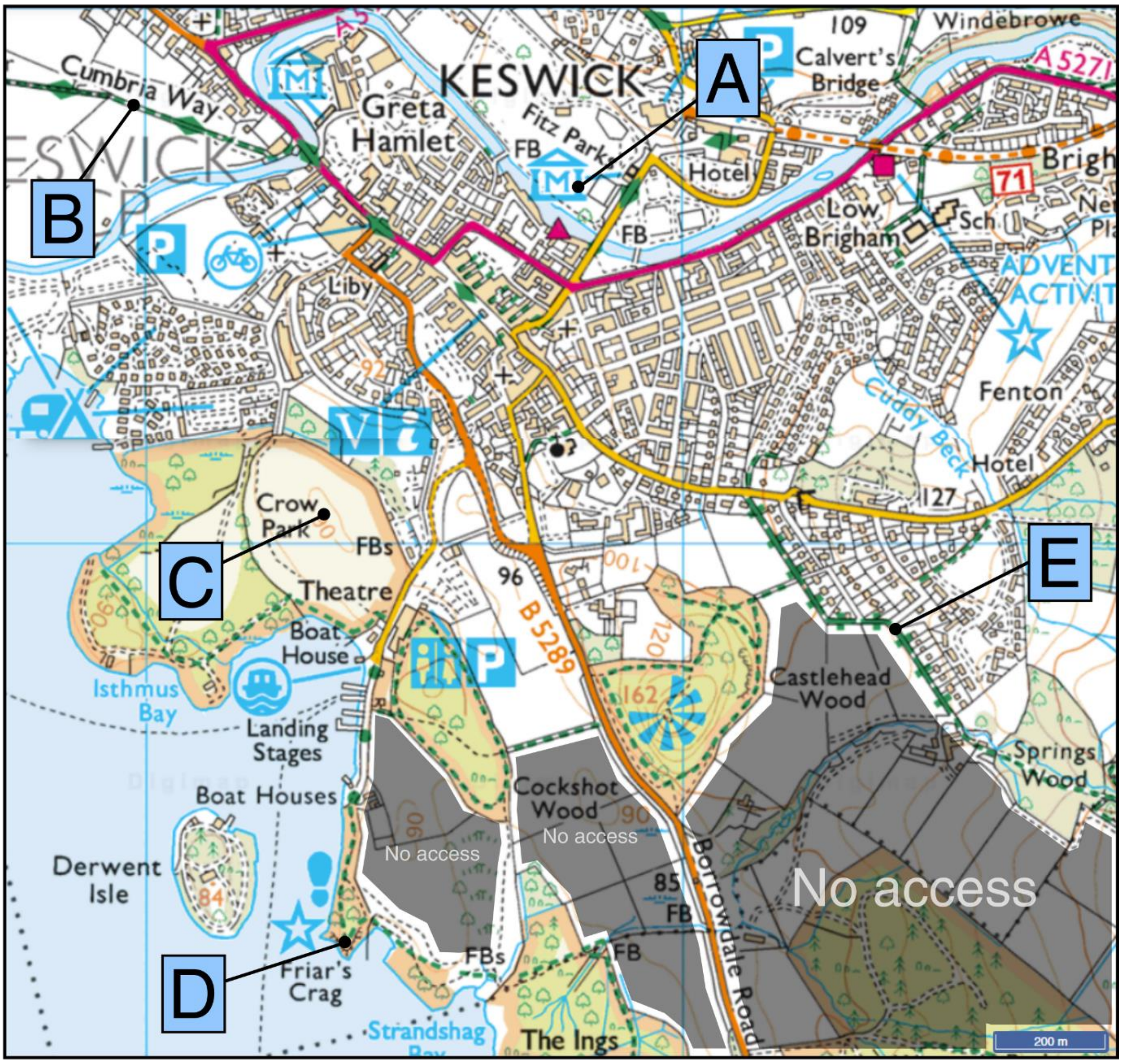
Each group should have multiple compasses and Abney levels, so take the opportunity to collect a number of measurements as a group to find the correct value. Try to be as precise as possible.

Competition

You will be in groups of 3 – 4. When you've completed your measurements, you should be able to estimate the elevations of peaks **I, II, III, IV** and **V** with some accuracy using the formulas shown on the previous pages. The group which gets closest to the true elevation of each peak will be the winner.

Prizes are yet to be determined, but as a minimum, this will be the respect and admiration of your fellow students.

Viewpoint locations: Keswick



Point A	54.60291, -3.13444, 87 m a.s.l
Point B	54.60407, -3.14751, 84 m a.s.l
Point C	54.59747, -3.14189, 96 m a.s.l
Point D	54.58995, -3.14089, 85 m a.s.l
Point E	54.59556, -3.12515, 119 m a.s.l

Point A (Fitz Park)

54.60291, -3.13444, 87 m

At Point A, measure the direction (compass) and vertical angle (Abney level) to Peak **I**.

Direction (°) to Peak **I** =

Vertical angle (°) to Peak **I** =

On page 15 onwards, we'll work through the calculations using measurements I've made previously, but collect your own here to compare the results.

Point B (Path to Portinscale)

54.60407, -3.14751, 84 m

At Point B, take measurements for Peaks **I, II** and **V:**

Direction (°) to Peak **I** =

Vertical angle (°) to Peak **I** =

Direction (°) to Peak **II** =

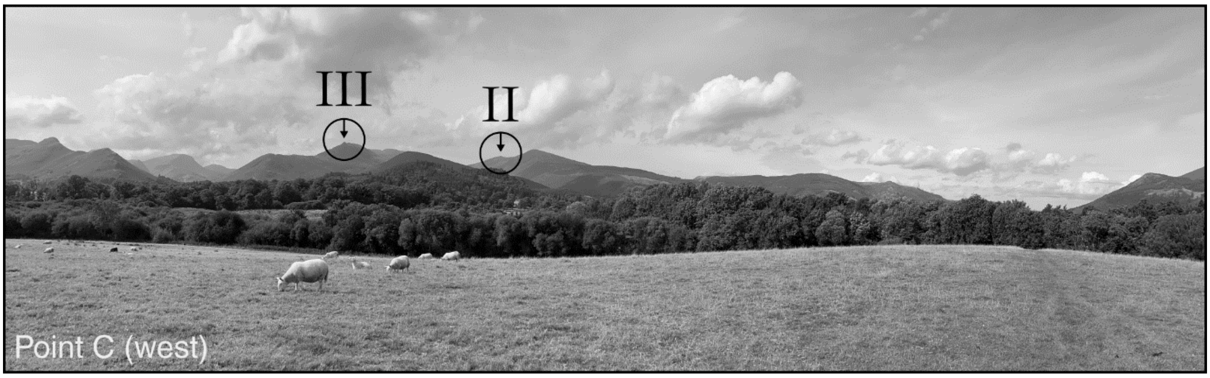
Vertical angle (°) to Peak **II** =

Direction (°) to Peak **V** =

Vertical angle (°) to Peak **V** =

Point C (Crow Park)

54.59747, -3.14189, 96 m



At Point C, take measurements for Peaks **II, III** and **IV:**

Direction (°) to Peak **II** =

Vertical angle (°) to Peak **II** =

Direction (°) to Peak **III** =

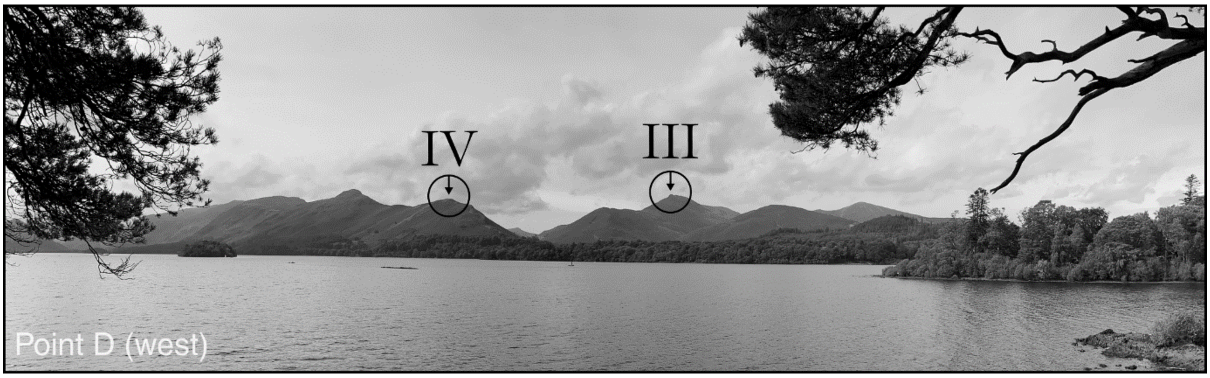
Vertical angle (°) to Peak **III** =

Direction (°) to Peak **IV** =

Vertical angle (°) to Peak **IV** =

Point D (Friar’s Crag)

54.58995, -3.14089, 85 m



At Point D, take measurements for Peak **III** (although others are visible):

Direction (°) to Peak **III** =

Vertical angle (°) to Peak **III** =

Point E (Springs Road)

54.59556, -3.12515, 119 m

At Point D, take measurements for Peaks **IV** and **V:**

Direction (°) to Peak **IV** =

Vertical angle (°) to Peak **IV** =

Direction (°) to Peak **V** =

Vertical angle (°) to Peak **V** =

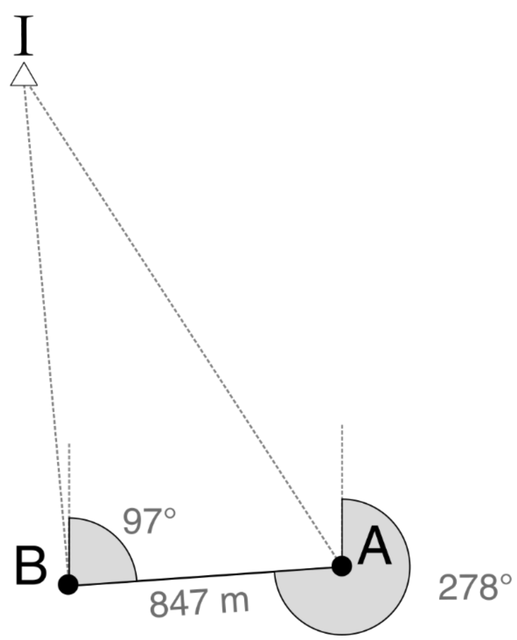
Calculations

If you're reading this, you should have visited all (or some) of the measurement points and made some measurements of your own. These following pages will you guide you through the steps required to convert these data into mountain peak elevations.

An example: Peak I

We'll go through the calculation steps together for Peak I, using some measurements I've collected earlier.

The diagram below shows that the distance between measurement points A and B is 855 m, with corresponding angles of 278° and 97°.

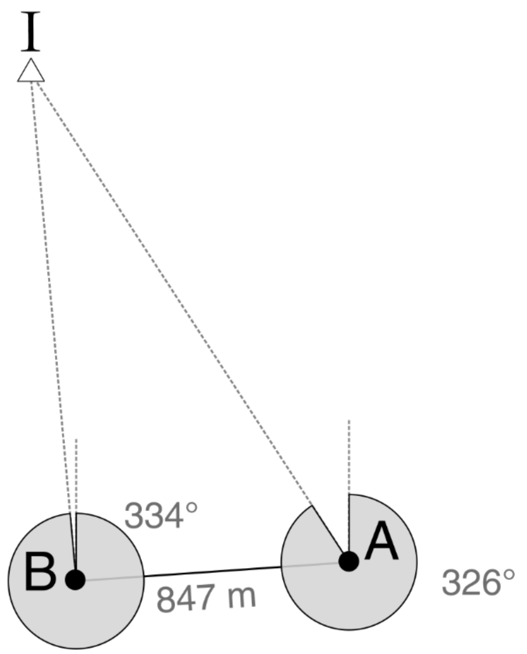


My results

My direction to Peak I was **326°** from Point A and **334°** from Point B.

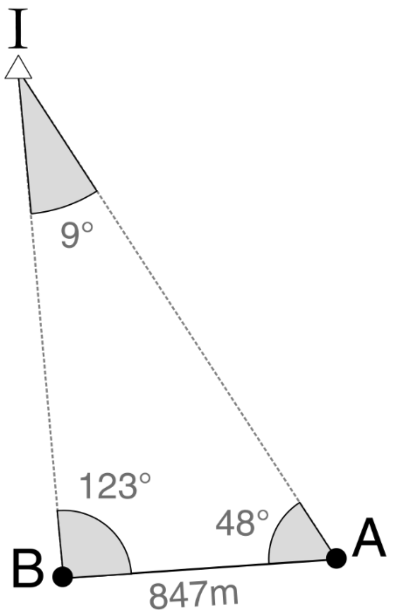
The vertical angles to Peak I was **5°** from Point A and **6°** from Point B.

NOT DRAWN TO SCALE



Using these measured directions, we can work out the interior angles of the triangle.

NOT DRAWN TO SCALE



Angle **A** = 326° – 278° = 48°

Angle **B** = 360° – 334° + 97°= 123°

Angle **C** = 180° – 123° – 48°= 9°

Using the **distance** formulas shown on p.3, we can now work out the length of the sides:

Side **A–I** = 847 m × sin(123) ÷ sin(9) = ~4541 m

Side **B–I** = 847 m × sin(48) ÷ sin(9) = ~4024 m

Using the **height** formulas shown on p.5, we can now work out the height of peak **I**, remembering to add on the elevations of the measurement points!

Height **I** (from **A**) = 4541 m × tan(5) + 87 m = 484.4 m

Height **I** (from **B**) = 4024 m × tan(6) + 84 m = 506.9 m

Result

The actual recorded height of the mountain is **502 m**, so we're off by approximately 18 m and 5 m respectively.

Q: Would you consider this to be a good result?

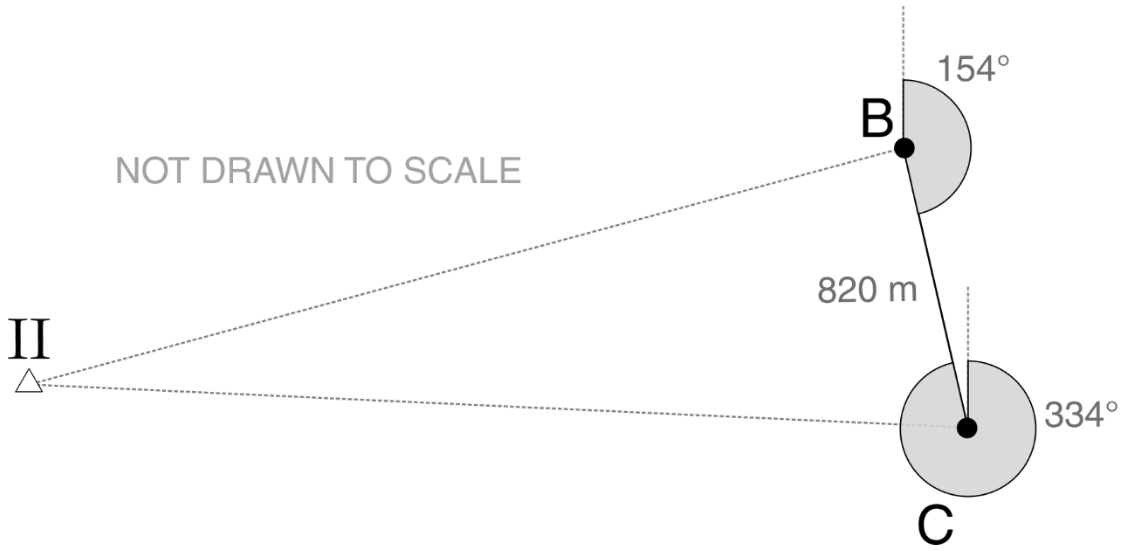
Q: What are the limitations of our approach?

Q: How could these be addressed?

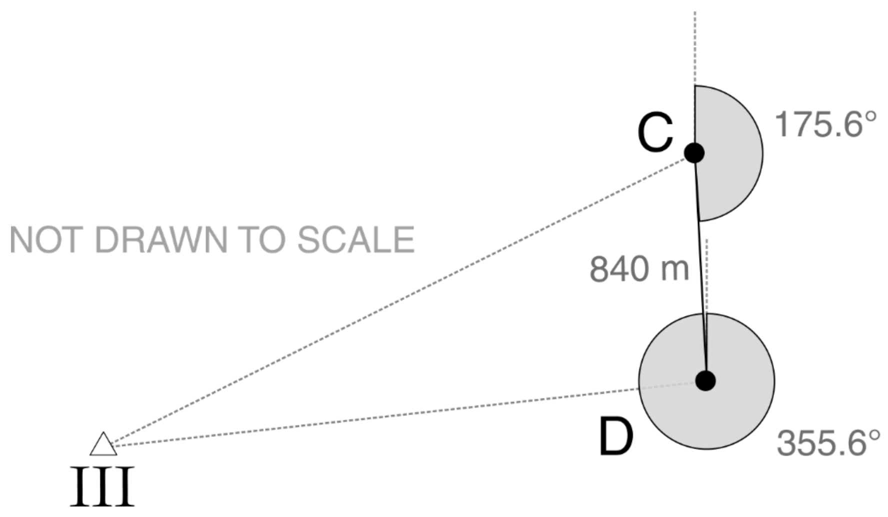
On the following pages, I've provided the necessary information to calculate the heights of peaks **II, III, IV** and **V.**

Have a go at calculating these.

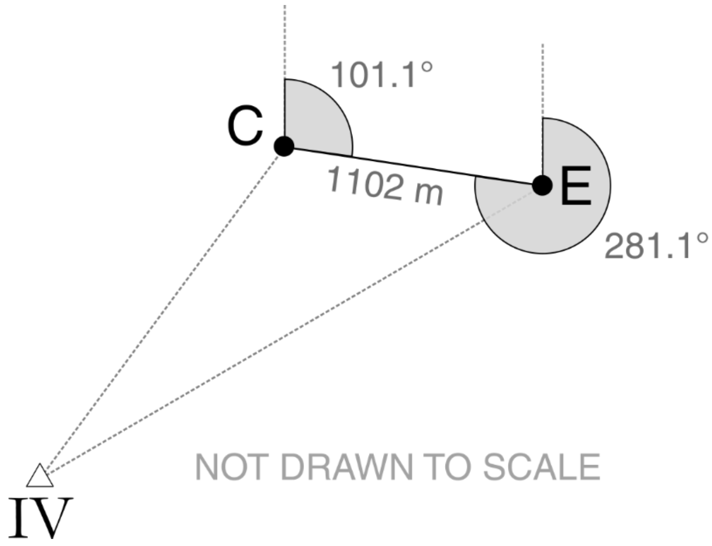
Peak II



Peak III



Peak IV



Peak V