

Tracing the Skyline

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Task

Using a compass (used for measuring horizontal angles) and Abney level (used for measuring vertical angles), estimate the elevations of the mountains surrounding the field centre.

Context

Mountains hold special scientific, cultural, and economic significance. Throughout history, they have been used to demarcate the boundaries between regions. More recently, **triangulation** from mountain summits has allowed us to estimate the size and dimensions of countries with much greater accuracy and precision e.g. the Principal Triangulation of Great Britain (1791 – 1853), as shown below:

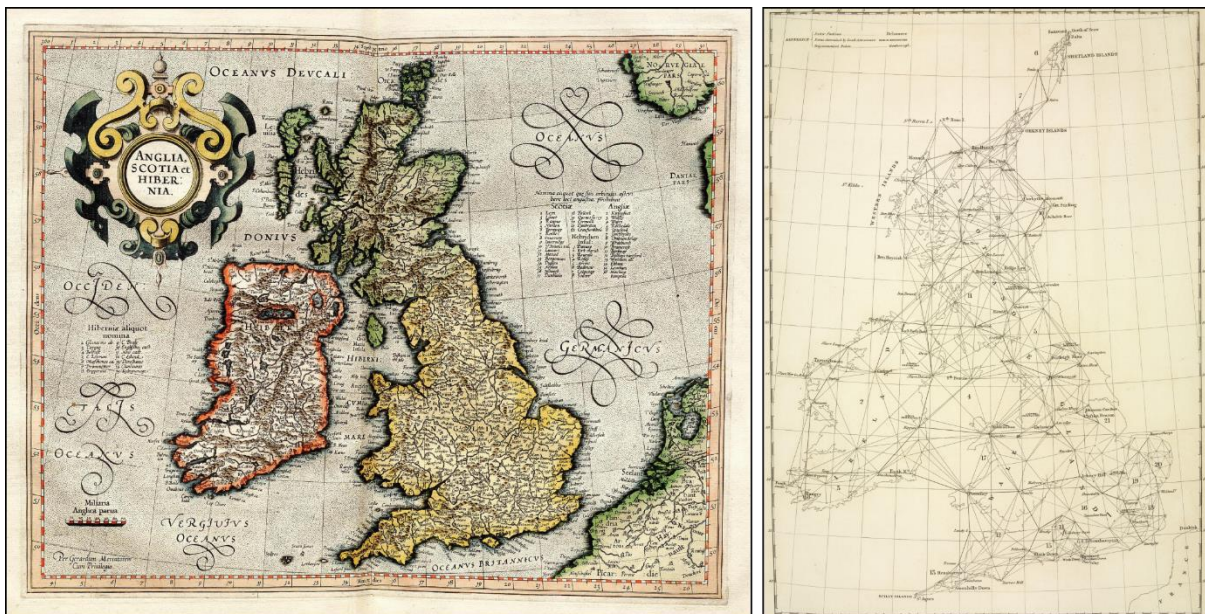


Figure 1: Maps of Great Britain and Ireland before triangulation (**left:** Atlas Cosmographicae by Gerardus Mercator, 1512 – 1594) and after triangulation (**right:** Principal Triangulation of Britain, 1791 – 1853). While triangulation was used for local mapmaking prior to this date, with the earliest evidence for the technique dating to ~1000 AD, this was the first attempt to apply the technique to an entire country.

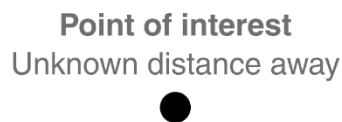
While mountains, coastlines and land areas are measured today using GPS and satellites, triangulation is still used for many purposes, including surveying, navigation, metrology etc. It is also an important component of **Geographic Information Science (GIS)**.

Finding the distance to objects

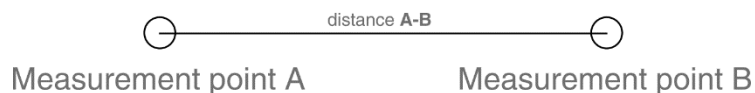
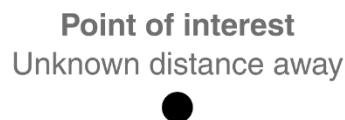
We can calculate the height of distant objects using **trigonometry**.

However, we first need to know the distance to them. This can be achieved by measuring the horizontal angles to that object from two (or more) other measurement points.

For example, let's say we are interested in a particular feature of the landscape (e.g. a mountain), but do not know how far away it is or its elevation. Here, it is represented as a single point:

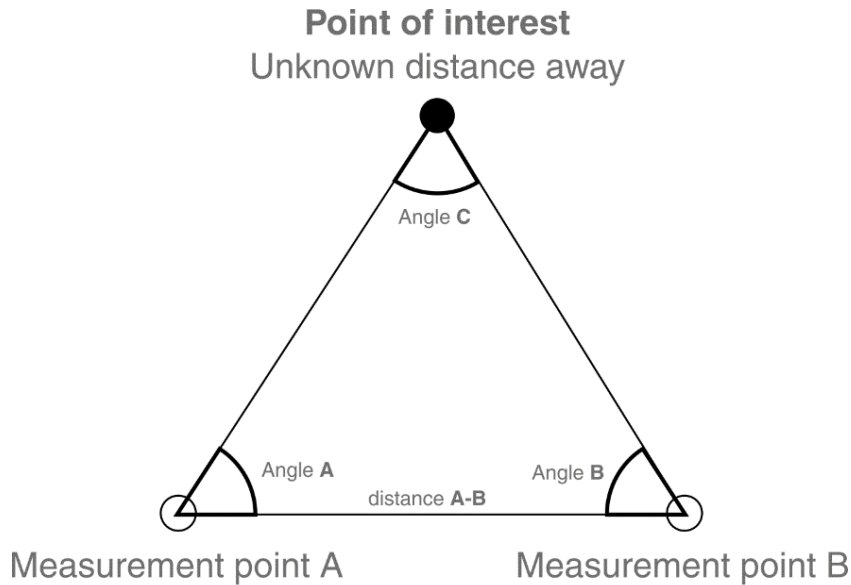


Next, we can select two measurement points (A and B) where we know the distance between them, as shown here:

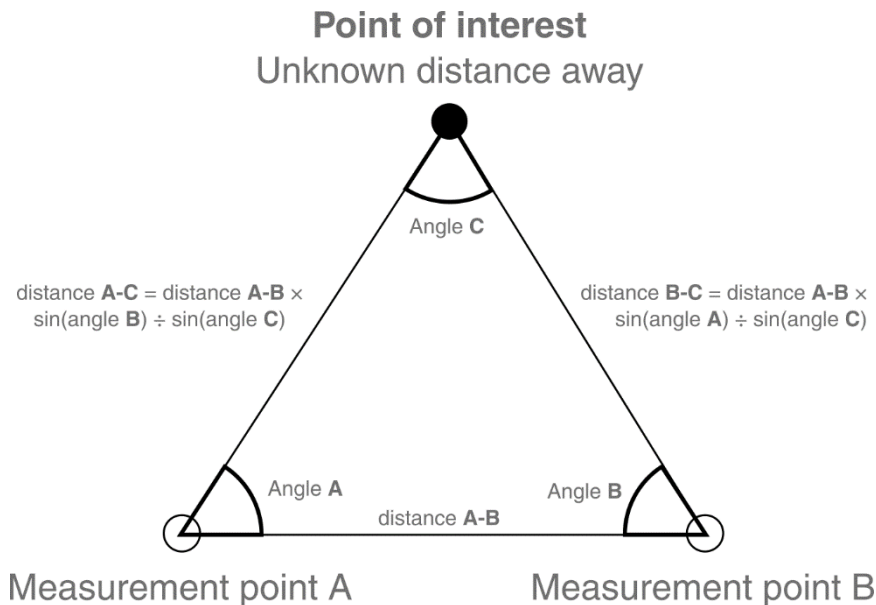


With this information, we now have everything we need to calculate the distance to the unknown point (and in future, it's height).

Our first step is to measure the horizontal angles from A and B to the measurement point using a compass. The final angle of the triangle (Angle C) equals $180 - \text{Angle A} - \text{Angle B}$, because the interior angles of a triangle = 180° .



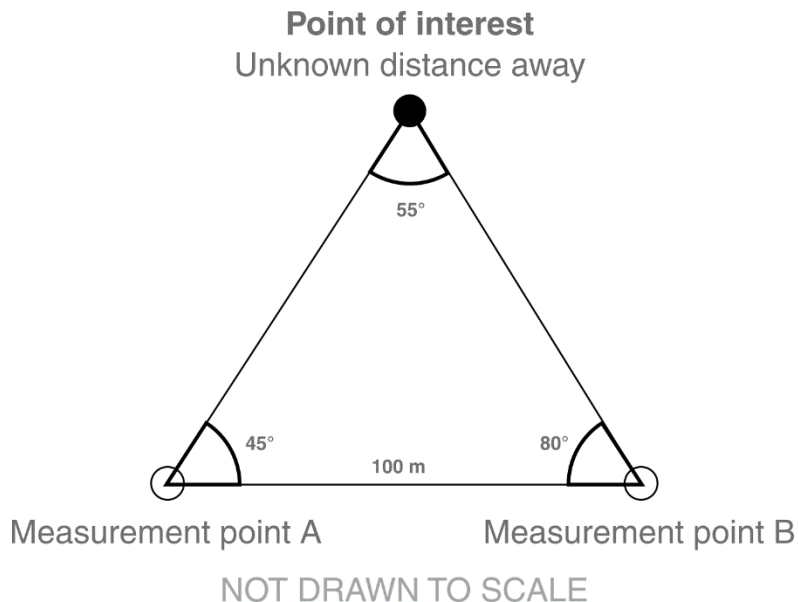
Using this information, the distances between A-C and B-C can be calculated using the trigonometric formulas shown below:



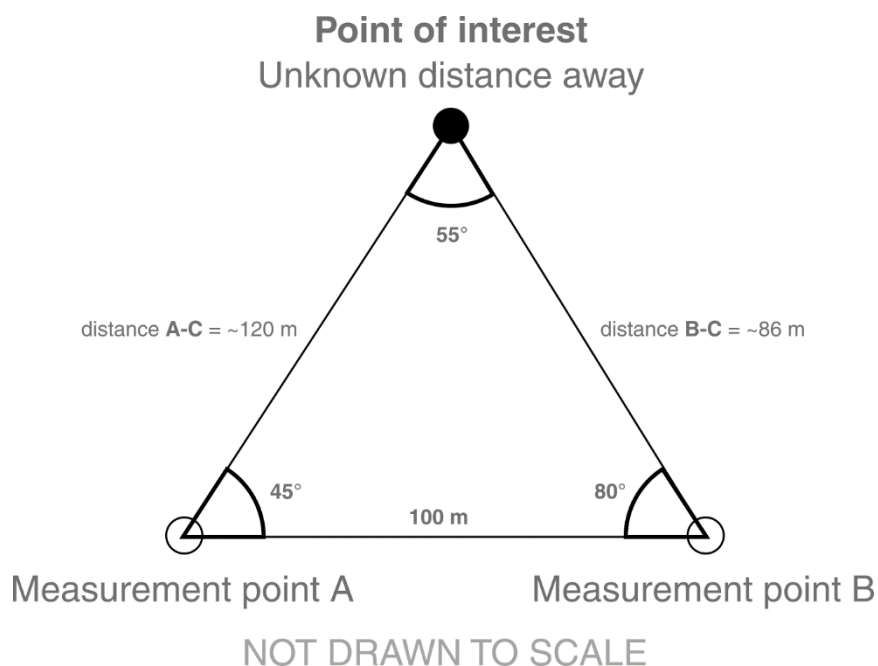
Lots of maths here, but **don't be alarmed**. We'll walk through a simple example on the next page.

Distance calculation: an example

As before, we are interested in a point of interest an unknown distance away and have selected two measurement points (A, B), which are 100 m apart. We have measured the angles from those points to the point of interest (45° , 80°), which leaves a final angle of 55° :



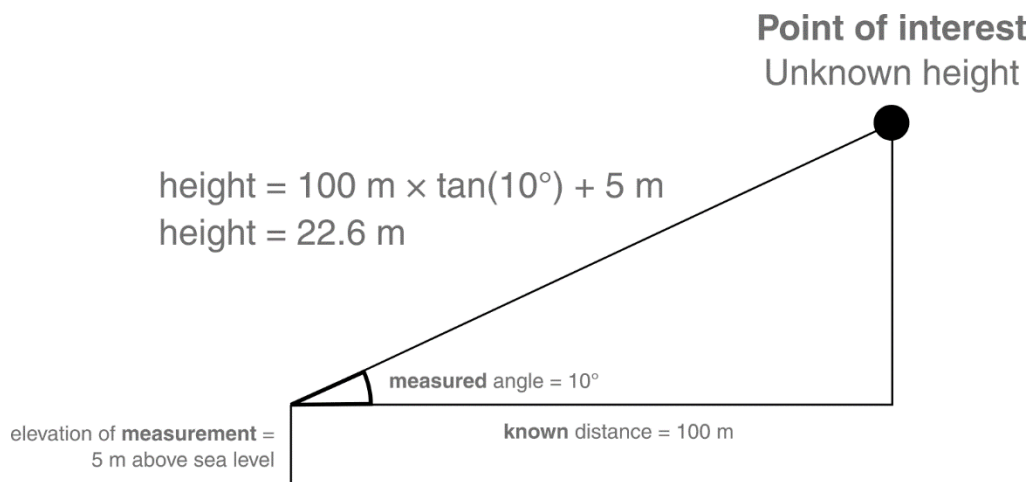
Using the formulas shown on the previous page (e.g. distance **B-C** = distance **A-B** \times $\sin(\text{angle A}) \div \sin(\text{angle C})$), the side lengths are returned as approximately 120 m and 86 m.



Finding the height of objects

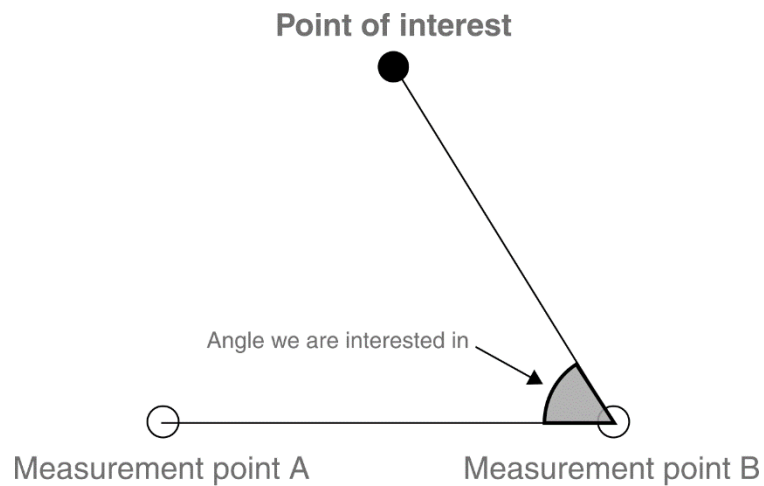
Once we know the distance to an object, finding its height is comparatively easy. This involves measuring the vertical angle to it (using the Abney level) and using the following formula:

$$\text{Height} = \text{distance} \times \tan(\text{angle}) + \text{elevation of the observer}$$

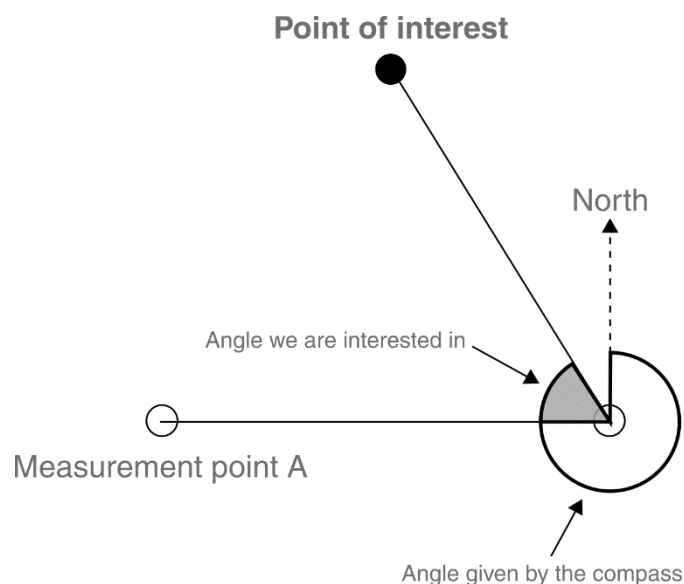


Compass directions vs. angles

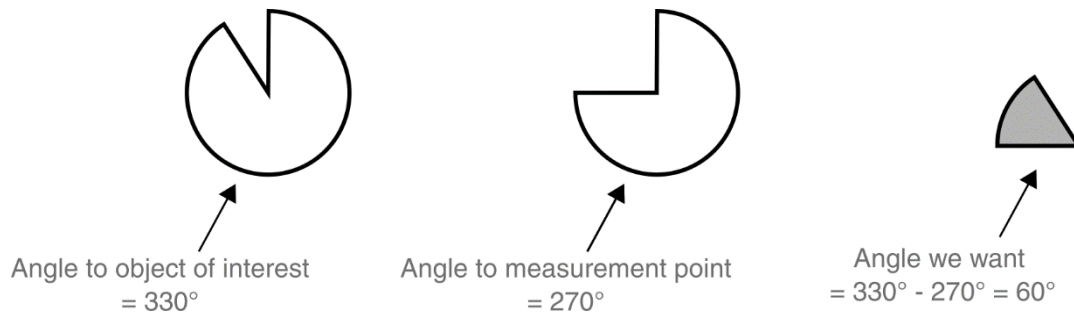
One difficult aspect of this technique is that the directions generated by a compass are not the same as the interior angles of a triangle. When we are measuring the directions, we are actually interested in the angle between our point of interest and the other measurement point, as shown below:



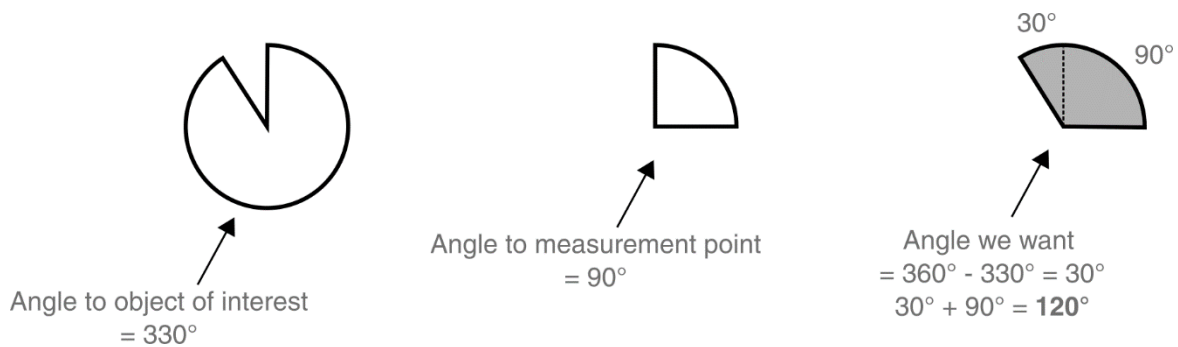
However, angles given by the compass are relative to magnetic north, so what we are actually measuring is the following:



We can retrieve the angle we are interested in if we know the directions between measurement points A and B (for this activity, these are provided for you). Based on the example above, the angle we need is simply the difference between the other two angles:



For other scenarios, including the reverse of the above, you will have to replicate the following:



Visualising the problem (as above) makes it much simpler to understand.

Putting theory into practice

With all this information at your disposal, your **task for today** is to visit five measurement points along a linear route from the field centre to Little Town (NY 234 195) and use a compass and Abney level to estimate the elevations of the surrounding mountains.

Measurement points are denoted as **A, B, C, D** and **E** and are shown on the map on the following page. You will be given a GPS to locate the exact locations.

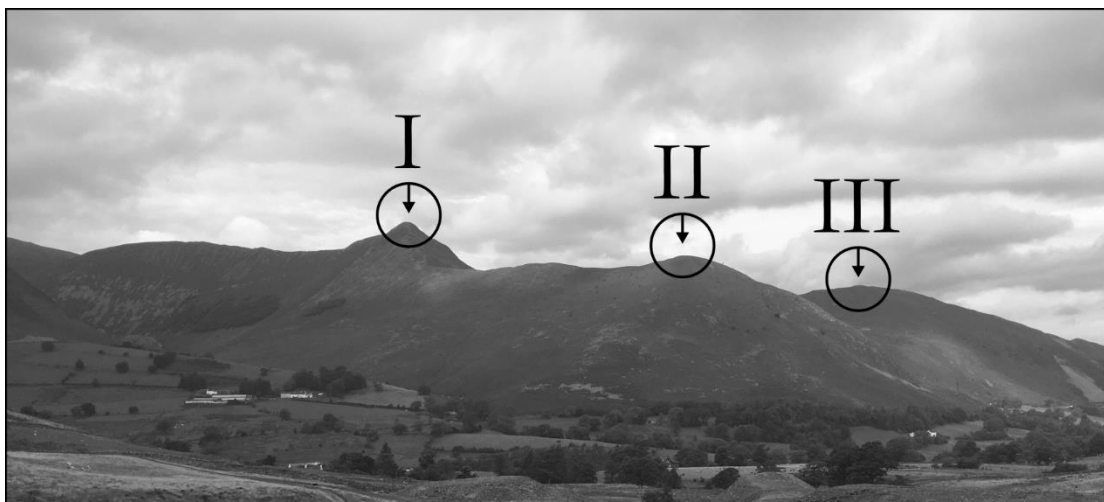
At each point, you'll need to measure the directions (compass) and vertical angles (Abney level) to nearby mountain summits.

Which summits?

As the Lake District weather can be fickle (i.e. low cloud, rain), **you can choose** whichever summits are visible on the day.

Aim to measure 4 – 5 summits and number them **I, II, III, IV** and **V** (to avoid confusion with the measurement points).

Note, ensure that each summit has been measured from two measurement points (the further apart the better) – you don't need to measure every peak from every measurement point. For example, below we may want to measure peaks **I, II** and **III**.



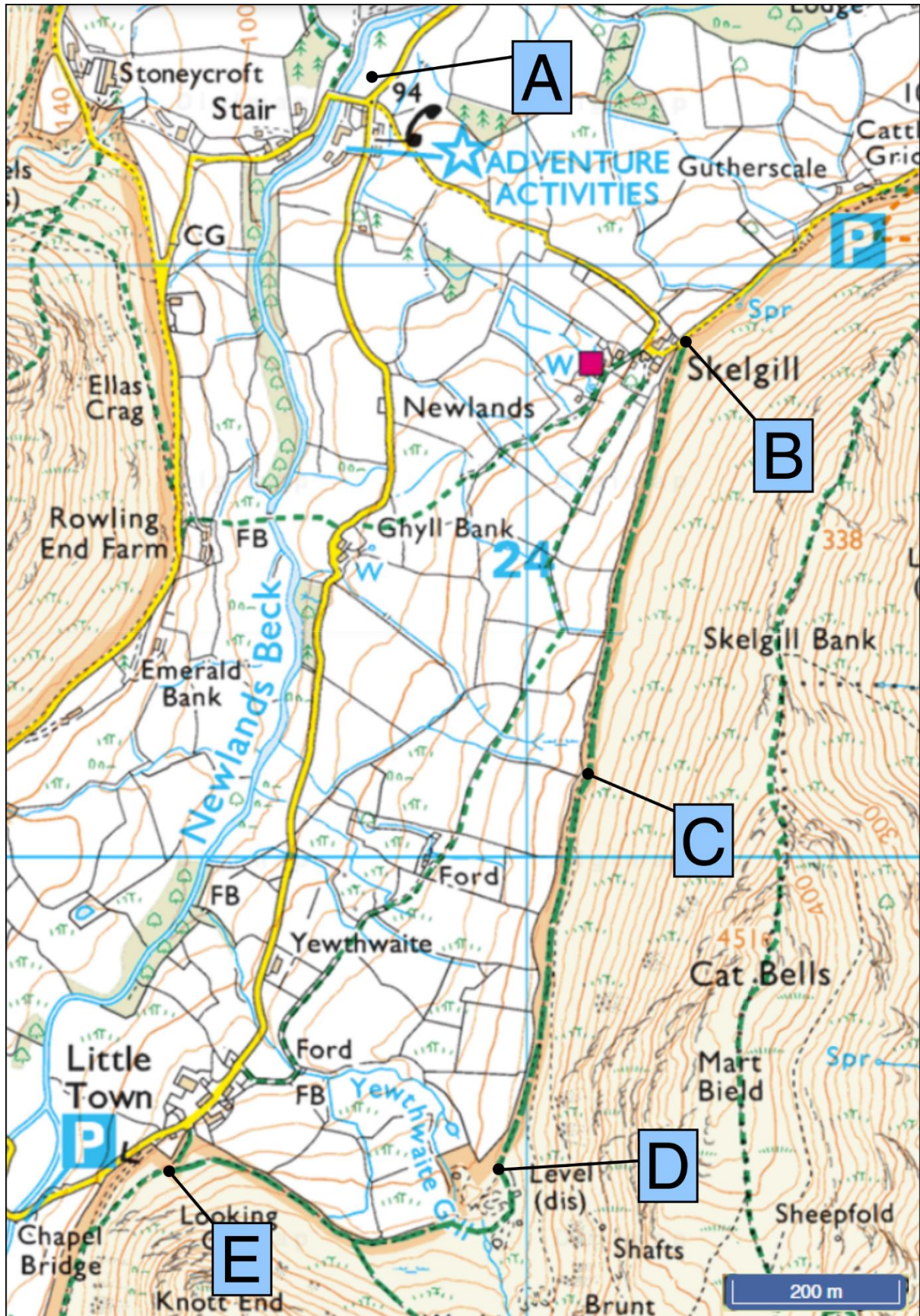
Each group should have multiple compasses and Abney levels, so take the opportunity to collect a number of measurements as a group to find the correct value. Try to be as precise as possible.

Competition

You will be in groups of 3 – 4. When you've completed your measurements, you should be able to estimate the elevations of the chosen peaks (I, II, III, IV and V) with some accuracy using the formulas shown on the previous pages. The group which gets closest to the true elevation of each peak will be the winner.

Prizes are yet to be determined, but as a minimum, this will be the respect and admiration of your fellow students.

Newlands measurement locations



Point A (Field centre)

54.58127, -3.18135, 94 m

Direction ($^{\circ}$) to Peak ___ =

Vertical angle ($^{\circ}$) to Peak ___ =

Direction ($^{\circ}$) to Peak ___ =

Vertical angle ($^{\circ}$) to Peak ___ =

Direction ($^{\circ}$) to Peak ___ =

Vertical angle ($^{\circ}$) to Peak ___ =

Direction ($^{\circ}$) to Peak ___ =

Vertical angle ($^{\circ}$) to Peak ___ =

Direction ($^{\circ}$) to Peak ___ =

Vertical angle ($^{\circ}$) to Peak ___ =

Remember, you don't have to measure every peak from every measurement point, but ensure you know which set of measurements belongs to which!

Point B (Skelgill)

54.57744, -3.17303, 158 m

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Point C (Path split)

54.57072, -3.17561, 212 m

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Point D (Path split)

54.56497, -3.17755, 222 m

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Point E (Path bend)

54.56465, -3.18634, 166 m

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Direction (°) to Peak ___ =

Vertical angle (°) to Peak ___ =

Calculations

If you're reading this, you should have visited all of the measurement points and made some measurements of your own. These following pages will you guide you through the steps required to convert these data into mountain peak elevations.

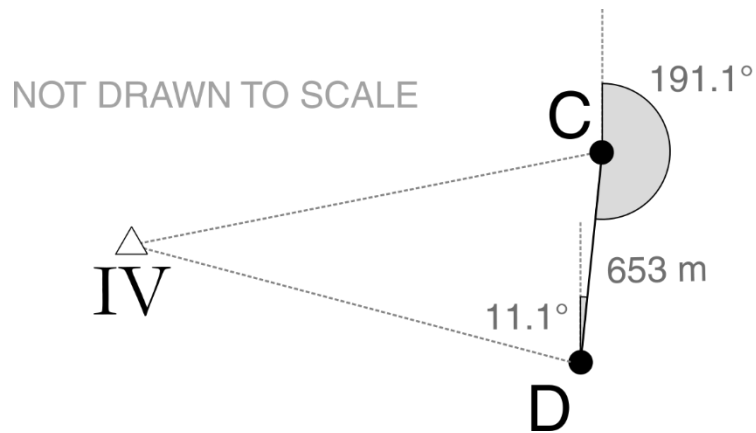
An example: Peak IV

We'll go through the calculation steps together for a hypothetical Peak IV, using some measurements I've collected earlier.



Peak IV was visible from a number of locations, but measurements were taken from points C and D (photo above from Keswick 2020).

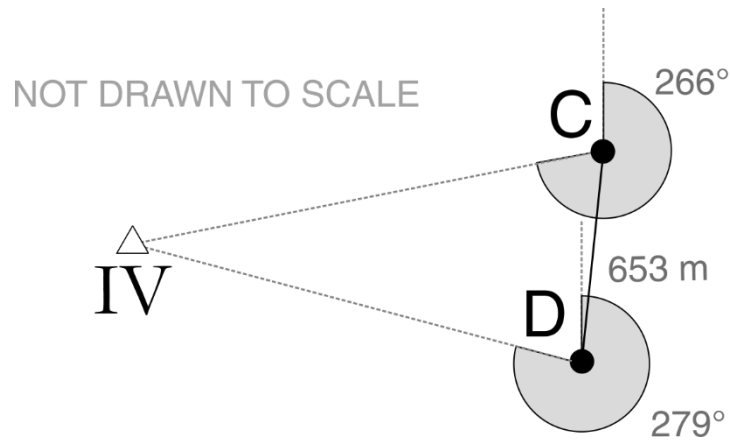
The diagram below shows that the distance between measurement points C and D is 653 m, with corresponding angles of 191.1° and 11.1° .



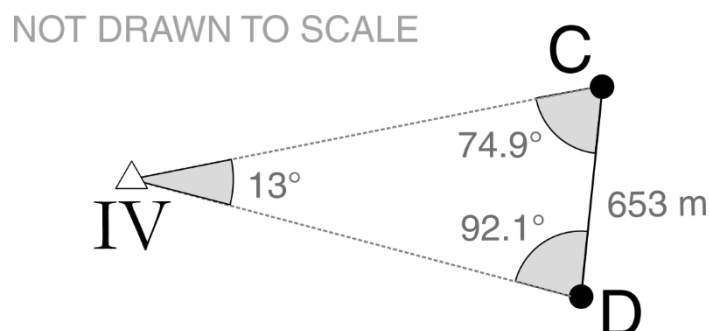
My results

My direction to Peak IV was 266° from Point C and 279° from Point D.

The vertical angles to Peak IV was 6.5° from Point C and 7° from Point D.



Using these measured directions, we can work out the interior angles of the triangle.



$$\text{Angle C} = 266^\circ - 191.1^\circ = 74.9^\circ$$

$$\text{Angle D} = 360^\circ - 279^\circ + 11.1^\circ = 92.1^\circ$$

$$\text{Angle IV} = 180^\circ - 74.9^\circ - 92.1^\circ = 13^\circ$$

Using the **distance** formulas shown on p.3, we can now work out the length of the sides:

$$\text{Side C-IV} = 653 \text{ m} \times \sin(74.9) \div \sin(13) = \sim 2802 \text{ m}$$

$$\text{Side D-IV} = 653 \text{ m} \times \sin(92.1) \div \sin(13) = \sim 2901 \text{ m}$$

Using the **height** formulas shown on p.5, we can now work out the height of peak I, remembering to add on the elevation of the measurement point!

$$\text{Height IV (from C)} = 2802 \text{ m} \times \tan(6.5) + 212 \text{ m} = 531.2 \text{ m}$$

$$\text{Height IV (from D)} = 2901 \text{ m} \times \tan(7) + 222 \text{ m} = 578.2 \text{ m}$$

Result

The actual recorded height of the feature is **555 m**, so we're off by approximately 20 – 30 m.

Q: Would you consider this to be a good result?

Q: What are the limitations of our approach?

Q: How could these be addressed?

Have a go at replicating the above approach for your chosen peaks. To do so, you'll need to know the distances and directions between each measurement point – included on the following page.

Measurement point information

The table below includes the distance between each measurement point (m), as well as the forward angle (e.g. A – B) and the backward angle (e.g. B – A).

Comparison	Distance (m)	Forward angle (°)	Back angle (°)
A – B	686	128.4	308.4
A – C	1231	162.4	342.5
A – D	1831	172.3	352.3
A – E	1878	189.9	9.9
B – C	766	192.6	12.6
B – D	1419	191.9	11.9
B – E	1664	211.2	31.1
C – D	653	191.1	11.1
C – E	969	225.8	45.7
D – E	570	266.4	86.4